Problem Set II

Macroeconomics II

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1 Dynamic Optimization with two periods ($T = 1$)

There is a representative household living for two periods ($t = 0, 1$). Let c_t and a_t denote the household's consumption and assets in period t, respectively. The household's objective function is given by

$$\max u(c_0) + \beta u(c_1),$$

where $u(c)$ is of the CIES form

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma},$$

$\sigma > 0, \sigma \neq 1$. Assuming that the household has no assets to start with ($a_0 = 0$), the dynamic budget constraints read

$$a_1 + c_0 = w_0,$$
$$a_2 + c_1 = w_1 + a_1 R_1.$$

R_t denotes the gross interest rate, w_t the wage in period t.

1. Discuss the conditions under which the household chooses $a_2 = 0$.
2. Setting $a_2 = 0$, combine the dynamic budget constraints to derive the intertemporal budget constraint (IBC). Interpret the IBC.
3. Setting $a_2 = 0$, solve the maximization problem of the household subject to either the dynamic budget constraints or the IBC. Derive and interpret the Euler equation.
4. According to the Euler equation, the slope of the consumption profile is determined by three factors: a consumption smoothing motive, patience, and intertemporal prices. Identify and interpret these factors. What happens to the consumption smoothing motive if the utility function is linear rather than strictly concave?
5. Use the Euler equation together with the intertemporal budget constraint to solve for the optimal consumption in periods $t = 0, 1$ (c_0^* and c_1^*) and the optimal savings at the end of period 0 (a_1^*).
6. The interest rate R_1 affects consumption threefold: through wealth, income and substitution effects. Identify and interpret these effects. What happens if the felicity function is logarithmic?

*I am sure there are many typos in the script. If you find any please send me an email to armando.naef@vwi.unibe.ch

2 Dynamic Optimization with many periods ($T > 1$)

There is a representative household living for $T+1$ periods ($t = 0, 1, \ldots, T$). The household's objective is to maximize the sum of discounted utility subject to a set of dynamic budget constraints, a terminal condition and an initial condition; i.e. the household's problem reads

$$\begin{aligned} \max \ & \sum_{t=0}^{T} \beta^t u(c_t) \\ \text{s.t. } & a_{t+1} = a_t R_t + w_t - c_t \text{ for } t = 0, 1, \ldots, T \\ & a_{T+1} \geq 0 \\ & a_0 R_0 \text{ given.} \end{aligned}$$

1. Derive the intertemporal budget constraint (IBC) by combining the dynamic budget constraints and the terminal condition ("conjecturing" $a_{T+1} = 0$). Interpret the IBC.
2. Solve the maximization problem subject to the IBC. Derive the Euler equation.
3. Suppose that $\beta R_t = 1$ for all t. Solve for the optimal consumption path. Do you need to make functional form assumptions about $u(\cdot)$?
4. Solve the maximization problem subject to the dynamic budget constraints and the terminal condition ($a_{T+1} \geq 0$) rather than the intertemporal budget constraint (maximization under inequality constraints). Derive the Euler equation and the transversality condition ($a_{T+1} = 0$).

3 Stochastic Consumption Path

Suppose that aggregate consumption is a stochastic variable. In particular, suppose that $\tilde{c}_t = \ln c_t$ has an i.i.d. Normal distribution with mean μ_c and variance σ_c^2. Suppose that the expected utility is given by

$$U = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma} \right],$$

where $\mathbb{E}_0[\cdot]$ is the expectations operator given the information set at period $t = 0$.

1. Why is there no difference between using $\frac{c_t^{1-\sigma}}{1-\sigma}$ as the felictiy function and $\frac{c_t^{1-\sigma}-1}{1-\sigma}$, except when $\sigma = 1$?
2. Derive an exression for U. *Hint:* if $\tilde{c}_t$ is normally distributed with mean μ_c and variance σ_c^2, then

$$\mathbb{E}\left[e^{\tilde{c}_t}\right] = e^{\mu_c + \frac{1}{2}\sigma_c^2}$$

3. The idea of welfare calculations is to investigate the impact of increases in σ_c keeping the value of $\mathbb{E}[c_t]$ the same. This means that we have to adjust μ_c if we change σ_c. Calculate the value for U when $\sigma_c = 0$ (i.e., no fluctuations) and when $\sigma_c = 0.02$. Let σ be equal to $0, 2,$ and 10. Remember to adjust μ_c.
4. The value of U is lower when σ_c is higher. Consider an agent facing $\sigma_c = 0.02$. With how much do I have to increase his consumption level each period to make him as well of as the agent facing $\sigma_c = 0$?

5. A value of $\sigma_c = 0.02$ is reasonable for aggregate fluctuations. The calculations above are based on the assumption that the risk is spread equally across the population. But now suppose that 95% of the population are never confronted with changes in their consumption level and all changes in aggregate consumption are due to 5% of the population adjusting their consumption level. Also, assume that the mean consumption level is the same across agents. How costly would business cycles be for those unlucky agents?