Problem Set II

Macroeconomics II

Solutions

Armando Näf*

October 7, 2020

1 Solution: Two periods ($T = 1$)

There is a representative household living for two periods ($t = 0, 1$). Let c_t and a_t denote the household's consumption and assets in period t, respectively. The household's objective function is given by

$$max\ u(c_0) + \beta u(c_1),$$

where $u(c)$ is of the CIES form

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma},$$

$\sigma > 0, \sigma \neq 1$. *Assuming that the household has no assets to start with ($a_0 = 0$), the dynamic budget constraints read*

$$a_1 + c_0 = w_0,$$
$$a_2 + c_1 = w_1 + a_1 R_1.$$

R_t denotes the gross interest rate, w_t the wage in period t.

1. *Discuss the conditions under which the household chooses $a_2 = 0$.*

   Solution:

   For any (c_0, a_1) a reduction in a_2 allows for higher consumption. This implies that the optimal choice for the household would be $a_2 \to -\infty$, because $u'(c_t) > 0,\ \forall c_t$. However, the economy does not exist in $t = 2$, therefore nobody will lend resources to the household at the end of $t = 1$. This implies that $a_2 \leq 0$. The two conditions imply that $a_2 = 0$.

$$\left.\begin{aligned} u'(c_t) > 0 \\ a_2 \leq 0 \end{aligned}\right\} \Rightarrow a_2 = 0 \tag{1}$$

2. *Setting $a_2 = 0$, combine the dynamic budget constraints to derive the intertemporal budget constraint (IBC). Interpret the IBC.*

   Solution:

*I am sure there are many typos in the script. If you find any please send me an email to armando.naef@vwi.unibe.ch

Use the dynamic budget constraint of the second period and solve for a_1

$$\underbrace{a_2}_{=0} + c_1 = w_1 + a_1 R_1$$
$$a_1 = \frac{c_1 - w_1}{R_1}. \tag{2}$$

Plug the result from equation (2) into the dynamic budget constraint of the first period:

$$a_1 + c_0 = w_0$$
$$\underbrace{\frac{c_1 - w_1}{R_1}}_{=a_1} + c_0 = w_0$$
$$c_0 + \frac{c_1}{R_1} = w_0 + \frac{w_1}{R_1}. \tag{3}$$

Consumption today and tomorrow depends on lifetime income. Realise that this is identical to a problem in microeconomics. However, instead of apple and oranges we have consumption today and tomorrow and the Relative price of consumption tomorrow is given by the inverse of the gross interest rate. Equivalently the endowment of the consumer is given by the net present value of today's and tomorrow's income.

3. *Setting $a_2 = 0$, solve the maximization problem of the household subject to either the dynamic budget constraints or the IBC. Derive and interpret the Euler equation.*

Solution:

- Let's first use the dynamic budget constraint to solve the household's problem:

$$\max_{c_0, c_1, a_1} u(c_0) + \beta u(c_1)$$
$$\text{s.t. } a_1 + c_0 = w_0$$
$$c_1 = w_1 + a_1 R_1$$

substitute, c_0, and c_1 by the corresponding dynamic budget constraint to get an unconstrained maximization problem with one choice variable:

$$\max_{a_1} u(w_0 - a_1) + \beta u(w_1 + a_1 R_1)$$

The first order condition is given by:

$$u'(\underbrace{w_0 - a_1}_{c_0})(-1) + \beta u'(\underbrace{w_1 + a_1 R_1}_{c_1}) R_1 = 0$$

This yields the familiar Euler equation

$$u'(c_0) = \beta R_1 u'(c_1) \tag{4}$$
$$\text{or } \frac{u'(c_0)}{\beta u'(c_1)} = R_1 \tag{5}$$
$$\tag{6}$$

where we substitute the consumption back in, to get equation (4). The first equation is referred to as the Euler equation, the second representation states the marginal rate of substitution between current and future consumption. The Euler equation describes optimal future consumption relative to current consumption. It determines the slope of the consumption path, which depends on a consumption smoothing motive, patience and intertemporal prices.

- Equivalently we could have derived the solution with the intertemporal budget constraint instead:

$$\begin{aligned} &\max_{c_0,c_1} u(c_0) + \beta u(c_1) \\ &\text{s.t. } c_0 + \frac{c_1}{R_1} = w_0 + \frac{w_1}{R_1} \end{aligned}$$

Where the FOC are given by the following equations:

$$\begin{aligned} c_0 &: u'(c_0) = \lambda \\ c_1 &: \beta u'(c_1) = \lambda \frac{1}{R_1} \end{aligned}$$

Combining the two equations then yields the Euler equation:

$$u'(c_0) = \beta R_1 u'(c_1)$$

4. *According to the Euler equation, the slope of the consumption profile is determined by three factors: a consumption smoothing motive, patience, and intertemporal prices. Identify and interpret these factors. What happens to the consumption smoothing motive if the utility function is linear rather than strictly concave?*

Solution:

To best identify the three different factors we should make use of the specific felicity function of the question. Realise that the Euler equation then takes the following form:

$$\begin{aligned} u'(c_0) &= \beta R_1 u'(c_1) \\ c_0^{-\sigma} &= \beta R_1 c_1^{-\sigma} \\ \frac{c_1}{c_0} &= (\beta R_1)^{\frac{1}{\sigma}} . \end{aligned} \tag{7}$$

To get to the last line in (7) we solved for the change in consumption, i.e. the slope of the consumption path, in dependence of the exogenous parameters.

This allows us to identy the three factors that determine the slope of the consumption path:

- Intertemporal price: The larger R_1, i.e. the relative price of consumption at $t = 1$ decreases, the steeper becomes the consumption path:

$$\frac{\partial c_o/c_1}{\partial R_1} = \frac{1}{\sigma} (\beta R_1)^{\frac{1}{\sigma}-1} \beta > 0$$

Intuitively if consumption at $t = 1$ gets cheaper, we want to decrease consumption today and increase consumption tomorrow.

- Patience: The larger β, i.e. the more patient a consumer is, the steeper becomes the consumption path:

$$\frac{\partial c_o/c_1}{\partial \beta} = \frac{1}{\sigma} (\beta R_1)^{\frac{1}{\sigma}-1} R_1 > 0$$

Intuitively if the consumer is more patient, she wants to decrease consumption today and increase consumption tomorrow which is relatively cheaper (relative price of consumption tomorrow is $\frac{1}{R_1} < 1$).

- Consumption smoothing motive given by $\frac{1}{\sigma}$

$$
\begin{aligned}
\frac{\partial(c_o/c_1)}{\partial(1/\sigma)} &= \frac{\partial \exp\left\{\frac{1}{\sigma}\ln(\beta R_1)\right\}}{\partial 1/\sigma} \\
&= \exp\left\{\frac{1}{\sigma}\ln(\beta R_1)\right\}\ln(\beta R_1)
\end{aligned}
$$

The consumption smoothing motive is best considered in three cases:

$$
\exp\left\{\frac{1}{\sigma}\ln(\beta R_1)\right\}\ln(\beta R_1)\begin{cases} >0, & \text{if } \beta R_1 > 1 \\ =0, & \text{if } \beta R_1 = 1 \\ <0, & \text{if } \beta R_1 < 1 \end{cases}
$$

If the felicity function is linear, rather than strictly concave then $\sigma = 0$ and the consumption smoothing motive goes to zero for all values of $\beta R_1 \neq 1$. In other words, the consumption path becomes infinitely steep.

5. *Use the Euler equation together with the intertemporal budget constraint to solve for the optimal consumption in periods $t = 0, 1$ (c_0^* and c_1^*) and the optimal savings at the end of period 0 (a_1^*).*

Solution:

We can use the Euler equation from (4) and make use of the felicity function from the question. This allows us to solve for c_1:

$$
\begin{aligned}
c_0^{-\sigma} &= \beta R_1 c_1^{-\sigma} \\
c_1 &= (\beta R_1)^{\frac{1}{\sigma}} c_0 \qquad (8)
\end{aligned}
$$

We use the result from (8) to replace c_1 in equation (3)

$$
\begin{aligned}
c_0 + \frac{c_1}{R_1} &= w_0 + \frac{w_1}{R_1} \\
c_0 + \frac{(\beta R_1)^{\frac{1}{\sigma}} c_0}{R_1} &= w_0 + \frac{w_1}{R_1} \\
c_0 &= \frac{w_0 + \frac{w_1}{R_1}}{1 + \frac{(\beta R_1)^{\frac{1}{\sigma}}}{R_1}} \qquad (9)
\end{aligned}
$$

Together with the previously derived result from (8) we can find the solution for c_1:

$$
c_1 = (\beta R_1)^{\frac{1}{\sigma}} \frac{w_0 + \frac{w_1}{R_1}}{1 + \frac{(\beta R_1)^{\frac{1}{\sigma}}}{R_1}} \qquad (10)
$$

Last, a_1 follows from the dynamic budget constraint of the first period:

$$
a_1 = w_0 - \frac{w_0 + \frac{w_1}{R_1}}{1 + \frac{(\beta R_1)^{\frac{1}{\sigma}}}{R_1}}. \qquad (11)
$$

6. *The interest rate R_1 affects consumption threefold: through wealth, income and substitution effects. Identify and interpret these effects. What happens if the felicity function is logarithmic?*

Solution:

We identify the three different effects with equation (9):

(a) Wealth Effect:

$$c_0 = \frac{w_0 + \frac{w_1}{\mathbf{R_1}}}{1 + \frac{(\beta R_1)^{\frac{1}{\sigma}}}{R_1}}$$

If $w_1 > 0$ an increase in the interest rate reduces wealth because it lowers the net present value of future income at $t = 0$, which lowers consumption in both periods (see (10)) - *negative wealth effect*

(b) Income Effect:

$$c_0 = \frac{w_0 + \frac{w_1}{R_1}}{1 + \frac{(\beta R_1)^{\frac{1}{\sigma}}}{\mathbf{R_1}}}$$

An increase in the interes rate, lowers the cost of the consumption bundle (c_0, c_1), which leads to a higher purchasing power for the consumer. This increases consumption in both periods - *positive income effect*

(c) Substitution Effect:

$$c_0 = \frac{w_0 + \frac{w_1}{R_1}}{1 + \frac{(\beta \mathbf{R_1})^{\frac{1}{\sigma}}}{R_1}}$$

An increase in the interest rate, lowers the relative price of c_1. Hence, it is optimal for the consumer to substitute some of the consumption today for consumption in the future. I.e. c_1 increases relative to c_0.

If the felicity function has a logarithmic form, then $\sigma = 1$. Equation (9) then becomes the following:

$$\begin{aligned} c_0 &= \frac{w_0 + \frac{w_1}{R_1}}{1 + \frac{(\beta R_1)^{\frac{1}{\sigma}}}{R_1}} \\ &= \frac{w_0 + \frac{w_1}{R_1}}{1 + \frac{\beta R_1}{R_1}} \\ &= \frac{w_0 + \frac{w_1}{R_1}}{1 + \beta}. \end{aligned}$$

Hence, the substitution and income effect cancel out and the interest rate effects consumption only through the wealth effect.

2 Solution: Dynamic Optimization with many periods ($T > 1$)

There is a representative household living for $T + 1$ periods ($t = 0, 1, \ldots, T$). The household's objective is to maximize the sum of discounted utility subject to a set of dynamic budget constraints, a terminal condition and an initial condition; i.e. the household's problem reads

$$\begin{aligned} & max \sum_{t=0}^{T} \beta^t u(c_t) \\ & s.t. \; a_{t+1} = a_t R_t + w_t - c_t \; for \; t = 0, 1, \ldots, T \\ & \quad a_{T+1} \geq 0 \\ & \quad a_0 R_0 \; given. \end{aligned}$$

1. *Derive the intertemporal budget constraint (IBC) by combining the dynamic budget constraints and the terminal condition ("conjecturing" $a_{T+1} = 0$). Interpret the IBC.*

   Solution:

   To derive the intertemporal budget constraint, we consider the dynamic budget constraint of the last period T and solve for the IBC at time 0 by backwards induction:

$$\begin{aligned} a_{T+1} &= a_T R_T + w_T - c_T \\ &= (a_{T-1}R_{T-1} + w_{T-1} - c_{T-1})\, R_T + w_T - c_T \\ &= ((a_{T-2}R_{T-2} + w_{T-2} - c_{T-2})\, R_{T-1} + w_{T-1} - c_{T-1})\, R_T + w_T - c_T \\ &= \dots \\ &= a_0 R_0 R_1 \cdots R_T + (w_0 - c_0) R_1 R_2 \cdots R_T + (w_1 - c_1) R_2 R_3 \cdots R_T \\ &\quad + \cdots + (w_{T-1} - c_{T-1}) R_T + (w_T - c_T) \end{aligned} \tag{12}$$

   let $q_t \equiv (R_1 R_2 \cdots R_t)^{-1}$ be the price of date-t consumption at the initial date and normalise $q_0 \equiv 1$. Multiplying the last equation by q_T then yields

$$\begin{aligned} q_T a_{T+1} &= a_0 R_0 + (w_0 - c_0) q_0 + (w_1 - c_1) q_1 + \cdots \\ &= a_0 + \sum_{t=0}^{T} q_t (w_t - c_t) \\ \Rightarrow \sum_{t=0}^{T} q_t c_t &= a_0 R_0 + \sum_{t=0}^{T} q_t w_t \end{aligned} \tag{13}$$

   Where (13) follows from the optimality condition $a_{T+1} = 0$. The IBC equates life time consumption spending and wealth.

2. *Solve the maximization problem subject to the IBC. Derive the Euler equation.*

   Solution:

$$\max_{\{c_t\}_{t=0}^{T}} \sum_{t=0}^{T} \beta^t u(c_t)$$
$$\text{s.t.} \sum_{t=0}^{T} q_t c_t = a_0 R_0 + \sum_{t=0}^{T} q_t w_t$$

   Forming the Lagrangian we get

$$\mathcal{L} = \sum_{t=0}^{T} \beta^t u(c_t) + \lambda \left[a_0 R_0 + \sum_{t=0}^{T} q_t w_t - \sum_{t=0}^{T} q_t c_t \right]$$

   Differentiating with respect to c_t yields the T first order conditions:

$$\beta^t u'(c_t) = \lambda q_t$$

   Using the first order condition for t and $t+1$ we can get rid of λ (note that $\lambda > 0$ due to local non satiation of the felicity function) and solve for the euler equation:

$$\beta^t u'(c_t) = \beta^{t+1}\frac{q_t}{q_{t+1}}u'(c_{t+1})$$
$$u'(c_t) = \beta R_{t+1} u'(c_{t+1})$$

where we make use of the fact that $\frac{q_t}{q_{t+1}} = \frac{R_1 R_2 \cdots R_t R_{t+1}}{R_1 R_2 \cdots R_{t-1} R_t} = R_{t+1}$

3. *Suppose that $\beta R_t = 1$ for all t. Solve for the optimal consumption path. Do you need to make functional form assumptions about $u(\cdot)$?*

   Solution:

   For $\beta R_t = 1$ it follows from the euler equation that $u'(c_t) = u'(c_{t+1}, \ \forall t$. If $u''(c_t) < 0$ then this implies a constant consumption path, i.e. $c_t = c_{t+1}$.

4. *Solve the maximization problem subject to the dynamic budget constraints and the terminal condition ($a_{T+1} \geq 0$) rather than the intertemporal budget constraint (maximization under inequality constraints). Derive the Euler equation and the transversality condition ($a_{T+1} = 0$).*

   Solution:

   If we use the T dynamic budget constraint, rather then the IBC the lagrangian takes the following form:

$$\mathcal{L} = \sum_{t=0}^{T} \left\{\beta^t u(c_t) + \lambda_t \left[a_{t+1} - (a_t R_t + w_t - c_t\right]\right\} + \mu a_{T+1}$$

   The first order conditions with respect to c_t and a_{t+1} are given by

$$\beta^t u'(c_t) = \lambda_t,$$
$$\lambda_t = \lambda_{t+1} R_{t+1}, \ t = 0, \dots, T-1$$
$$\lambda_t = \mu, \ t = T$$
$$\mu a_{T+1} = 0$$

   where the last line corresponds to the complementary slackness condition. From the first equation we find that $\lambda_t > 0, \ \forall t$ due to local non satiation of the felicity function. This implies that $\mu > 0$ and hence $a_{T+1} = 0$ the *transversality condition* because of the complementary slackness condition. Further we find the euler equation by combining the first and the second line of the first order condition:

$$\beta^t u'(c_t) = \lambda_{t+1} R_{t+1}$$
$$= \beta^{t+1} R_{t+1} u'(c_{t+1})$$
$$u'(c_t) = \beta R_{t+1} u'(c_{t+1})$$

3 Solution: Stochastic Consumption Path

Suppose that aggregate consumption is a stochastic variable. In particular, suppose that $\tilde{c}_t = \ln c_t$ has an i.i.d. Normal distribution with mean μ_c and variance σ_c^2. Suppose that the expected utility is given by

$$U = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}\right],$$

where $\mathbb{E}_0[\cdot]$ is the expectations operator given the information set at period $t = 0$.

1. *Why is there no difference between using* $\frac{c_t^{1-\sigma}}{1-\sigma}$ *as the felictiy function and* $\frac{c_t^{1-\sigma}-1}{1-\sigma}$*, except when* $\sigma = 1$*?*

   Solution:

   The felicity function can be split into two seperate parts: $\frac{c_t^{1-\sigma}-1}{1-\sigma} = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{-1}{1-\sigma}$. It is clear that the second part of this function is simply a shifter of the utility level as it does not depend on the consumption level and does therefore not affect the marginal utility of the household. However, the level of utility (which is shifted by the second term) does not contain any information. It follows that the additional term has no impact on the problem except for $\sigma = 1$, when it is needed to apply *l'Hôpital's rule.*

2. *Derive an expression for* U*. Hint: if* $\tilde{c}_t$ *is normally distributed with mean* μ_c *and variance* σ_c^2*, then*

$$\mathbb{E}\left[e^{\tilde{c}_t}\right] = e^{\mu_c + \frac{1}{2}\sigma_c^2}$$

   Solution:

$$\begin{aligned}
U &= \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \frac{c_t^{1-\sigma}-1}{1-\sigma}\right] \\
&= \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \frac{(e^{\tilde{c}_t})^{1-\sigma}-1}{1-\sigma}\right] \\
&= \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \frac{e^{(1-\sigma)\tilde{c}_t}-1}{1-\sigma}\right] \\
&= \sum_{t=0}^{\infty}\beta^t \frac{\mathbb{E}_0\left[e^{(1-\sigma)\tilde{c}_t}\right]-1}{1-\sigma}
\end{aligned}$$

   Note: If $x \sim N(\mu_x, \sigma_x^2)$ then $\alpha x \sim N(\alpha\mu_x, \alpha^2\sigma_x^2)$ where α is a constant. It follows that

$$\begin{aligned}
U &= \sum_{t=0}^{\infty}\beta^t \frac{\mathbb{E}_0\left[e^{(1-\sigma)\tilde{c}_t}\right]-1}{1-\sigma} \\
&= \sum_{t=0}^{\infty}\beta^t \frac{\left[e^{(1-\sigma)\mu_c + \frac{(1-\sigma)^2}{2}\sigma_c^2}\right]-1}{1-\sigma} \\
&= \frac{\left[e^{(1-\sigma)\mu_c + \frac{(1-\sigma)^2}{2}\sigma_c^2}\right]-1}{(1-\sigma)(1-\beta)}
\end{aligned}$$

3. *The idea of welfare calculations is to investigate the impact of increases in* σ_c *keeping the value of* $\mathbb{E}[c_t]$ *the same. This means that we have to adjust* μ_c *if we change* σ_c*. Calculate the value for* U *when* $\sigma_c = 0$ *(i.e., no fluctuations) and when* $\sigma_c = 0.02$*. Let* σ *be equal to* $0, 2,$ *and* 10*. Remember to adjust* μ_c*.*

   Solution:

   When doing a welfare analysis we want to make sure that the expected consumption level remains the same, while increasing the size of the business cycles. This allows us to determine how costly different sizes of business fluctuations are. Let μ_c^* be the value for μ when $\sigma_c = 0$. Recall that

$\mathbb{E}[c_t] = \mathbb{E}\left[e^{\tilde{c}_t}\right] = e^{\mu_c + \frac{1}{2}\sigma_c^2}$. Thus, if we want that $\mathbb{E}[c_t]$ does not depend on σ_c, then we need that

$$\mu_c = \mu_c^* - \frac{1}{2}\sigma_c^2$$

To make explicit, that U depends on σ_c and μ_c^*, we denote the households utility by $U(\mu_c^*, \sigma_c)$. If $\sigma_c = 0$ then it follows that

$$U(\mu_c^*, 0) = \frac{e^{(1-\sigma)\mu_c^*} - 1}{(1-\sigma)(1-\beta)}$$

Equivalently, when $\sigma_c > 0$ then

$$\begin{aligned} U(\mu_c^*, \sigma_c) &= \frac{e^{(1-\sigma)\mu_c + \frac{(1-\sigma)^2}{2}\sigma_c^2} - 1}{(1-\sigma)(1-\beta)} \\ &= \frac{e^{(1-\sigma)\left(\mu_c^* - \frac{1}{2}\sigma_c^2\right) + \frac{(1-\sigma)^2}{2}\sigma_c^2} - 1}{(1-\sigma)(1-\beta)} \end{aligned}$$

It follows that the utility of an agent is lower when σ_c is higher

$$\begin{aligned} U(\mu_c^*, \sigma_c) &< U(\mu_c^*, 0) \\ \frac{e^{(1-\sigma)\left(\mu_c^* - \frac{1}{2}\sigma_c^2\right) + \frac{(1-\sigma)^2}{2}\sigma_c^2} - 1}{(1-\sigma)(1-\beta)} &< \frac{e^{(1-\sigma)\mu_c^*} - 1}{(1-\sigma)(1-\beta)} \\ e^{(1-\sigma)\left(\mu_c^* - \frac{1}{2}\sigma_c^2\right) + \frac{(1-\sigma)^2}{2}\sigma_c^2} &< e^{(1-\sigma)\mu_c^*} \\ (1-\sigma)\left(\mu_c^* - \frac{1}{2}\sigma_c^2\right) + \frac{(1-\sigma)^2}{2}\sigma_c^2 &< (1-\sigma)\mu_c^* \\ \frac{(1-\sigma)^2}{2}\sigma_c^2 &< (1-\sigma)\left(\frac{1}{2}\sigma_c^2\right) \\ 1-\sigma &< 1. \end{aligned}$$

In other words, we find that the utility is higher as long as $\sigma > 0$ and equal if $\sigma = 0$, which implies that as long as the consumer is strictly risk avers, the utility is higher for a a lower σ_c.

4. *The value of U is lower when σ_c is higher. Consider an agent facing $\sigma_c = 0.02$. With how much do I have to increase his consumption level each period to make him as well of as the agent facing $\sigma_c = 0$?*

   Solution:

   To find an expression, for how much we need to compessate the consumer for an increase in risk, we start with our previous result and let Γ be the value by which we must increase the mean of the distribution of $\tilde{c}_t$:

$$\begin{aligned}
U(\Gamma + \mu_c^*, \sigma_c) &= U(\mu_c^*, 0) \\
\frac{e^{(1-\sigma)\left(\Gamma+\mu_c^*-\frac{1}{2}\sigma_c^2\right)+\frac{(1-\sigma)^2}{2}\sigma_c^2}-1}{(1-\sigma)(1-\beta)} &= \frac{e^{(1-\sigma)\mu_c^*}-1}{(1-\sigma)(1-\beta)} \\
e^{(1-\sigma)\left(\Gamma+\mu_c^*-\frac{1}{2}\sigma_c^2\right)+\frac{(1-\sigma)^2}{2}\sigma_c^2} &= e^{(1-\sigma)\mu_c^*} \\
(1-\sigma)\left(\Gamma+\mu_c^*-\frac{1}{2}\sigma_c^2\right)+\frac{(1-\sigma)^2}{2}\sigma_c^2 &= (1-\sigma)\mu_c^* \\
\Gamma - \frac{1}{2}\sigma_c^2 + \frac{(1-\sigma)}{2}\sigma_c^2 &= (0 \\
\Gamma &= \sigma\frac{1}{2}\sigma_c^2
\end{aligned}$$

Therefore, if $\sigma_c = 0.02$, then we must increase the expected value of $\tilde{c}_t$ by 0.0002σ, which is equivalent to raising aggregate consumption by $0.02\sigma\%$ (because $\tilde{c}_t$ is in logs, a shifter in its mean is approximately equal to a percentage change of the variable in levels).

5. *A value of $\sigma_c = 0.02$ is reasonable for aggregate fluctuations. The calculations above are based on the assumption that the risk is spread equally across the population. But now suppose that 95% of the population are never confronted with changes in their consumption level and all changes in aggregate consumption are due to 5% of the population adjusting their consumption level. Also, assume that the mean consumption level is the same across agents. How costly would business cycles be for those unlucky agents?*

   Solution: To find an answer to this question we must figure out how large σ_c is for the unlucky agents. From the question we know that

   $$c_t = \phi\bar{c} + (1-\phi)c_{u,t}$$

   where $\bar{c}$ is the constant consumption of the lucky agents and $c_{u,t}$ the consumption of the unlucky agents. This implies that

   $$\frac{c_t - \bar{c}}{\bar{c}} = (1-\phi)\frac{c_{u,t}-\bar{c}}{\bar{c}},$$

   which implies that the standard deviation of percentage changes in $c_{u,t}$ is $\frac{1}{1-\phi}$ times the standard deviation of percentages changes in aggregate consumption. An aggregate $\sigma_c = 0.02$ implies that the standard deviation of $\tilde{c}_{u,t}$ is equal to 0.4, given that $\phi = 0.95$. This implies that the increase from $\sigma_c = 0$ to $\sigma_c = 0.02$ corresponds to a loss in consumption for the unlucky agent by $\sigma\frac{1}{2}0.4^2$, which is equal to a loss in consumption by 16% for the unlucky agents if they have a moderate risk aversion given by $\sigma = 2$.