Problem Set IV

Macroeconomics II

Armando Näf*

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1 General Equilibrium Conditions

Consider a a neoclassical growth model (Ramsey model) with infinitely lived agents. Households have utility such that $u'(\cdot) > 0$ and $u''(\cdot) > 0$ and firms have a constant return to scale production function.

1. State the conditions that arise from
   (a) utility maximization of households,
   (b) profit maximization of firms, and
   (c) feasibility.
2. Combine these conditions to derive the two core equations that characterize the representative agent model.

2 General Equilibrium: Steady state

Consider the infinitely lived representative agent model analyzed in exercise 1. Suppose that utility is of the CRRA type,

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma},$$

and production is characterized by a Cobb-Douglas function,

$$f(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha},$$

or, in per capita terms,

$$f(k_t, 1) = k_t^{\alpha}.$$

1. Derive and plot the steady state resource constraint and the steady state Euler equation (with k on the horizontal axis of the diagram and c on the vertical axis).
2. Solve for the steady state values of consumption and capital (the *modified-golden-rule* capital stock).
3. Derive the *golden-rule* capital stock. Show that the modified-golden-rule capital stock is necessarily smaller than the golden-rule capital stock.
4. Why does steady state consumption fall short of consumption at the golden-rule capital stock (although the equilibrium is Pareto efficient)?

*I am sure there are many typos in the script. If you find any please send me an email to armando.naef@vwi.unibe.ch

3 General Equilibrium: Phase diagram and model dynamics

Consider the neoclassical growth model from exercise 2.

1. Show the dynamics within the phase diagram and draw the saddle path.
2. Suppose that the economy is in the steady state. How do k_{t+1}, c_t, w_t and R_t respond to the following, somewhat model-inconsistent shocks?
   (a) An earthquake destroys some of the initial (steady state) capital stock.
   (b) There is a onetime, permanent increase in technology a, i.e. the production function changes from $f(k_t, 1)$ to $a \cdot f(k_t, 1)$, with $a > 1$.

   Draw the adjustment path. Moreover, explain the adjustments from the household's point of view.