

Problem Set V

Macroeconomics II

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1 General equilibrium in an OLG Model

Consider a standard OLG model we studied in class with two cohorts, *young* and *old*. Households are infinitely lived with utilities such that $u'(\cdot) > 0$ and $u''(\cdot) < 0$. Firms have access to a constant return to scale technology.

1. Assume that the population is constant. State the conditions that arise from
 - (a) utility maximization of households,
 - (b) profit maximization of firms, and
 - (c) feasibility.
2. Combine these conditions to derive the three core equations that characterize the OLG model without population growth.

2 Law of motion for capital, equilibrium allocation and prices

Consider an OLG model like the one we discussed in 1 without population growth. Assume logarithmic preferences $u(c_t) = \ln(c_t)$.

1. Show that the equilibrium law of motion for capital is given by $k_{t+1} = \frac{\beta}{1+\beta} f_L(k_t, 1)$.
2. Characterize the equilibrium path of k_t graphically assuming the Cobb-Douglas production function $f(K_t, L_t) = K_t^\alpha L_t^{1-\alpha}$.
3. Characterize the equilibrium allocation and prices.

3 Individual and aggregate savings

Suppose that young households save a constant fraction of their labor income for retirement. Assume that there is a constant population growth ν , such that $N_{1,t} = \nu N_{2,t}$, where $N_{i,t}$ is the size of cohort i at time t .

1. Define savings of young and old households as well as aggregate savings.
2. How do aggregate savings evolve over time if the population size is either constant or growing and wages are either constant or growing?

*I am sure there are many typos in the script. If you find any please send me an email to armando.naef@vwi.unibe.ch

4 Government intervention

Consider an infinite-horizon OLG model without population growth in which capital does not contribute to production (i.e., $f(K_t, L_t) = L_t$ or $f(k_t, 1) = 1$), but may be stored from one period to the next with depreciation rate δ .

1. Show that the steady state of this economy is necessarily dynamically inefficient.

Suppose that there is a government transferring b from young to old households in each period.

2. Which condition needs to hold for a marginal increase in b leading to a Pareto improvement? Does it hold for the economy in this exercise?
3. What amount of social security transfer b is optimal?