Problem Set VI

Macroeconomics II

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1 Incomplete markets

There is a representative household living for three periods ($t = 0, 1, 2$). There is only one asset available for saving. The household faces risk regarding future wages and interest rates (i.e., w_1, R_1, w_2 and R_2 are unknown). Suppose that there are two possible states of nature in periods $t = 1$ and $t = 2$, state h and state l, which occur with probabilities π_h and $\pi_l = 1 - \pi_h$, respectively.

1. Write down the maximization problem of the household. What are the choice variables in this problem?
2. Solve the maximization problem and derive the three Euler equations.

Suppose now additionally that $\beta R_t = 1$ for all t, there are no assets to start with ($a_0 = 0$), and utility is quadratic, i.e.,

$$u(c_t) = \phi c_t - \frac{1}{2} c_t^2,$$

in which ϕ is large relative to c_t. Wages in period $t = 1$ and $t = 2$ can take the values $w_{1,i}$ and $w_{2,ij}$, respectively, where i indexes the state in period $t = 1$ and j indexes the state in period $t = 2$, with $i, j \in \{h, l\}$.

3. Show that the quadratic utility function features decreasing, linear marginal utility.
4. Derive the intertemporal budget constraint(s). How many of them are there?
5. Use the Euler equations to show that $\mathbb{E}_0 [c_1 - c_0] = 0$.
6. Using the intertemporal budget constraints as of time $t = 0$ and $t = 1$, derive the optimal consumption c_0^*, $c_{1,h}^*$ and $c_{1,l}^*$.
7. What is the sign and the magnitude of $c_{1,i}^* - c_0^*, i = h, l$?

2 Complete markets

Consider a two-period model with two states of nature in the second period, h and l, occurring with probabilities π_h and $\pi_l = 1 - \pi_h$ and in which wages amount to $w_1(h)$ and $w_1(l)$, respectively. There are two assets, $a_1^{(1)}$ and $a_1^{(2)}$. In period $t = 0$ the household starts with wage w_0 and initial assets a_0.

*I am sure there are many typos in the script. If you find any please send me an email to armando.naef@vwi.unibe.ch

1. Suppose that the return vectors of the assets are given by $[1\ r]$ and $[s\ 1]$, respectively, across the two states h and l. If $r = s = 0$, these assets are Arrow securities. However, Arrow securities are not required for market completeness. Are markets complete if $r = s = 1$? What if $r = 1$ and $s = 0.5$? What if $r = \frac{1}{s}$?
2. Write down the dynamic budget constraints given return vectors $[R_1^{(1)}(h)\ R_1^{(1)}(l)]$ and $[R_1^{(2)}(h)\ R_1^{(2)}(l)]$.
3. Suppose that the return vectors are $[R_1^{(1)}(h)\ 0]$ and $[0\ R_1^{(2)}(l)]$. Solve the maximization problem of the household and derive the Euler equations.